UNCERTAINTY IN TRAFFIC FORECASTS: LITERATURE REVIEW AND NEW RESULTS FOR THE NETHERLANDS

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Abstract. This paper provides a review of transport model applications that not only provide a central traffic forecast (or forecasts for a few scenarios), but also quantify the uncertainty in the traffic forecasts in the form of a confidence interval or related measures. Both uncertainty that results from using uncertain inputs (e.g. on income) and uncertainty in the model itself are treated. The paper goes on to describe the methods used and the results obtained for a case study in quantifying uncertainty in traffic forecasts in The Netherlands.

1. INTRODUCTION

Transport models are regularly used in many countries around the world to predict the international, national, regional or local transport volumes and traffic flows on specific network links for a single scenario or a limited number of scenarios. The same models are also used to give the likely impacts of transport infrastructure projects (e.g. new roads, wider roads, new railway lines) and transport policies (e.g. road pricing). All these predictions are point estimates, and, even when produced for several scenarios, do not give insight into the uncertainty margin that exists around these forecasts.

However, for decision-making on infrastructure projects and transport policy measures it is very important to have an estimate not only of the most likely outcome, but also to know the possible range of future values for the transport volumes and the probabilities attached to these possible outcomes. It might be better to invest in a project that on average is slightly less profitable, but considerably less risky in terms of the variation in future traffic volumes, than in a more profitable, risky project. Quantifying uncertainty in traffic forecasts can therefore lead to better-informed decision-makers and better decision-making

Although thousands of papers on transport model forecasts can be found in journals, conference proceedings and reports, the literature on quantifying uncertainty in traffic forecasts is fairly limited.

In this paper we present an overview of the literature on uncertainty in transport modelling. Further, we provide the outcomes of an application in The Netherlands.

The word 'uncertainty' can mean many things. It can refer to (Klir and Wierman, 1999; Klir, 2006):

- Discord or dispersion, due to evidence supporting mutually exclusive alternatives;
- Non-specificity or imprecision, due to evidence supporting nested alternatives;
- Fuzziness or vagueness, due to evidence not supporting sharp definition of alternatives.

This paper is about traffic forecasts that are uncertain not because their definitions are fuzzy (these are sharply defined), but because the values these variables will take in the future are unknown. There are several (many) mutually exclusive values that these variables can take, each with a certain probability (and these probabilities sum to one; Klir and Folger, 1988). In terms of the classification above, we are thus dealing with dispersion.

A transport model consists of equations combining exogenous variables (also called 'inputs') and coefficients (also called 'parameters') that express how the endogenous variables (or 'outputs'), such as travel demand or link flows, depend on the exogenous variables (and possibly also on other endogenous variables). In forecasting for future years, forecasts for the exogenous variables from other sources, usually other models, are inserted into the model. Uncertainty in the model outputs can be due to:

- Input uncertainty: the future values of the exogenous variables (e.g. the future incomes) are unknown;
- Model uncertainty:
    - Specification error in the model equations (omitted variables, inappropriate assumptions on functional form and statistical distributions for random components);
    - Error due to using parameter estimates instead of the true values.

Section 2 of this paper presents the main outcomes of the literature review on quantifying the amount of uncertainty in forecasting with transport models. In section 3, the method we developed for the treatment of input uncertainty as well as for model uncertainty is described. Section 4 contains outcomes for an application of this method in The Netherlands. Finally, in section 5 the conclusions from this investigation are listed.

2. THE LITERATURE REVIEW

A review of the literature on quantifying uncertainty in transport models was carried out. For each paper or report reviewed, we were seeking answers to the following questions:

1. What type of uncertainty has been studied (uncertainty due to model inputs, the model itself or both)?
2. For which variables is uncertainty studied (e.g. link flows, value of time)?
3. Which methods have been used for quantifying uncertainty in traffic forecasts?
4. How is uncertainty expressed?
5. What is the order of magnitude of the uncertainty around the forecasts?

In our review of the international literature, we did not find a large number of publications on calculating uncertainty measures for transport model forecasts, presumably because this topic has not been studied frequently. In Table 1 below, the outcomes from the literature review are summarised.

This overview leads to the following observations and conclusions.

Type of uncertainty studied

Seven of the 21 studies that we identified investigate both model and input uncertainty, nine study only model uncertainty (mainly in the model coefficients, usually not specification errors) and four restrict themselves to input uncertainty (one study did not distinguish between model and input uncertainty). It is remarkable that two-thirds of the studies focus on one specific kind of uncertainty. Especially for papers on model uncertainty, the research question often is not how large the total uncertainty in the outcomes is, but has to do with the consequences of decisions to be taken on model structure and sample size.

Variables for which uncertainty is studied

Only six studies out of the 21 listed have quantified the impact on link flows, which is the key output of traffic models that is used in project evaluation. However, several other papers dealt with uncertainty in revenues, travel times and emissions, which are usually derived by combining the predicted link flows with further information (e.g. fees, fares, speed-flow curves, emission factors). Seven studies produce uncertainty for a more aggregate output than link flows: total area-wide demand for one or more modes, measured in trips, passenger-kilometres and/or vehicle-kilometres. A few studies are not really looking at travel demand and related outputs, but only at uncertainty in time and costs coefficients or in the value of time (which is a ratio of model coefficients).

Table 1. Summary of the literature on uncertainty of traffic forecasts (pkm: passenger kilometres; vkm: vehicle kilometres; vmt: vehicle miles travelled).

Publication	Type of uncertainty studied	Variables for which uncertainty is studied	Methods to quantify uncertainty	How is uncertainty expressed	Order of magnitude of uncertainty
Ashley, 1980	Model and input uncertainty	Traffic flow on specific (old and new) road links	Random draws from distributions for inputs and model coefficients	Graph of probability distribution of traffic flows	Probability of 5% that flow on new by-pass will be less than 18,000 vehicles/16 hours and 5% that it will be more than 36,000
Lowe, et al., 1982	Input uncertainty (focus) and model uncertainty	Link flows	Random draws from distributions for inputs and model coefficients	Percentiles	Probability of 5% that flow will be less than 14,000 vehicles/day and 5% that it will be more than 20,000
Ben-Akiva and Lerman, 1985	Model uncertainty	Transport cost and time coefficients (as an example)	Analytic formula for model uncertainty in multi-coefficients model	95% confidence interval	Not reported
De Jong, 1989	Model uncertainty (sampling, coefficients)	Number of households with a car; number of car km/year	Analytic formula for sampling and estimation variance	Variation and standard error	Estimation standard error between 3% and 6% of predicted values
Fowkes, 1995	Model uncertainty (coefficients)	Coefficients of modal split model, including costs, wait time and in-vehicle time; Willingness to pay	Repeated estimation on simulated datasets	Standard deviation of estimated coefficients; Confidence interval around mode benefit	95% confidence interval for mode benefit ranges from 0 to twice the average value.
Kroes, 1996	Input uncertainty and model uncertainty (incl. model application)	Link flows and revenues	Repeated model runs for simulated inputs and coefficients	Standard error and other statistics	Not reported
Leurent, 1996	Input uncertainty	Travel time; Daily number of cars on a link	Repeated model runs for simulated inputs	Standard error	Standard deviation is about 10% of predicted flow

Publication	Type of uncertainty studied	Variables for which uncertainty is studied	Methods to quantify uncertainty	How is uncertainty expressed	Order of magnitude of uncertainty
Brundell-Freij, 1997	Model uncertainty	Coefficients of modal split model, including costs, time and constants	Repeated estimation on simulated datasets for different sample sizes	t-ratios and confidence intervals for estimated coefficients	Even with 850 observations 5 out of 11 parameters are not significant.
De Jong et al, 1998	Model uncertainty (specification, coefficients)	Value of time	Jack-knife method and draws from multivariate normal distribution	Standard error	Standard deviation between 6% and 24% of average values of time
Boyce, 1999	Model and input uncertainty (focus on inputs)	vkm	Repeated model simulation, drawing from distributions for input variables	Standard errors and ratio of forecasts	Not reported
Grue, 1999	Model uncertainty	Number of cars, number of trips by mode, elasticities	Repeated model simulation, drawing from distributions for model coefficients	95% confidence interval	95% confidence interval ±1% for set of income coefficients on the total number of short trips and ±8% for long trips
Brundell-Freij, 2000	Model uncertainty (specification, sampling, estimation)	Value of time	As Brundell-Freij, 1997; Bootstrap analysis	Standard error of the value of time	Standard error between 3% and 20% of in-vehicle value of time
Zhao and Kockelman, 2002	Model and input uncertainty	Link flows	Random draws for inputs and parameters in 4-stage model	Standard error	Uncertainty propagates when going from trip generation to distribution and modal split, but is reduced in assignment.
Rodier and Johnston, 2002	Input uncertainty	Trips, vmt, vehicle hours delay, (emissions)	Sensitivity analysis on number of input factors	Percentage over- and under-prediction	0-70% under- or overprediction
Armoogum, 2003	Model and input uncertainty	Number of trips and pkm	Jack-knife and scenario analysis	Variance and percentage deviation from reference	Model uncertainty: for trips in 2030 variance is 27% of the mean (pkm: 6%).

Publication	Type of uncertainty studied	Variables for which uncertainty is studied	Methods to quantify uncertainty	How is uncertainty expressed	Order of magnitude of uncertainty
Boyce and Bright, 2003	Model and input uncertainty (focus on inputs)	Revenue from privately-financed project	Repeated model simulation, drawing from distributions for input variables; Scenario analysis	Percentiles; private funders want to see 95-99% probability of no loss.	Only the worst scenario fell below the first percentile
Ecorys, 2003	Input uncertainty	Revenues	Sensitivity analysis	Different revenue amounts	Revenues 1.2 or 3.9 mln depending on traffic growth
Ministerie van Financiën and CPB, 2003	No distinction made between model and input uncertainty	Financial outcomes of projects	Add a risk paragraph in project assessment	Not reported	Not reported
Research Results Digest, 2003	Model uncertainty (coefficients)	Number of pavement sections	Jack-knife method	Correlation coefficient, standard error	Not reported
Schrijver et al., 2003	Input uncertainty	Travel time	Random draws from inputs distributions	Interval around mean travel time	Not reported
Beser Hugosson, 2004, 2005	Model uncertainty (coefficients)	Total and OD demand by mode, link flows, train lines and value of time	Bootstrap sampling, repeated estimation and model application	95% confidence interval	95% confidence interval between ±7% and ±14%

Methods for reflecting the impact of input uncertainty on forecast uncertainty

We split up the discussion on the methods to quantify uncertainty in traffic forecasts in two parts: one on the effect of input uncertainty on output uncertainty and one on the impact of model uncertainty on output uncertainty.

All methods encountered in the literature for quantifying the amount of input uncertainty use some form of repeated model simulation (sensitivity testing). The same model is applied over and over again, with different inputs. A commonly used method for generating different inputs is scenario analysis. However, in scenario analysis no probabilities are attached to the various scenarios under study. This makes calculation of overall standard errors or related uncertainty measures for the model outcomes impossible. Many of the studies investigated postulate statistical distributions for the input variables and then draw (usually at random, sometimes at specific percentiles) input values from these distributions. The resulting values are then used in model runs. Final outcomes for uncertainty are calculated from the variance over all the runs for the different input values. This seems to be the standard approach to produce input uncertainty.

Most studies use univariate distributions for the input variables; correlation between inputs is ignored (unlike scenario studies that try to sketch consistent futures). More realistic estimates of uncertainty can be derived if one takes account of correlations between inputs (e.g. income and car ownership) by drawing from multivariate distributions, but this requires knowledge on the correlations. Lowe et al. (1982) used an experimental design (as in an SP survey) on the input variation, which can increase the efficiency of the process of running the model (not all combinations are needed).

Methods for reflecting the impact of model uncertainty on forecast uncertainty

For quantifying model uncertainty in transport forecasts, we find a wider diversity of approaches than for input uncertainty. Some studies used analytic expressions for the variance of the endogenous variable that result from using parameter estimates for the influence of the exogenous variables. This can only be done if the model equations are relatively straightforward. For more complicated models, these expressions become very cumbersome and often only approximations (e.g. from Taylor series expansion) can be given.

To obtain proper t-ratios or standard errors for the model coefficients in situations with specification error (such as repeated measurements in panel and SP data), the Jack-knife method and the related Bootstrap method are sometimes used (see Cirillo et al., 2000). In the Jack-knife, sub samples are created from an original sample by systematically omitting a small fraction of the data. The Bootstrap is applied by sampling at random from the original sample with replacement.

After having calculated the proper standard errors for the parameters, these can be used either in an analytic calculation of the standard error (due to estimation) of the model outcomes or as information on the statistical distribution of the parameters of the model, from which values can be drawn for model simulation runs, similarly to the method used for input uncertainty.

Again, it is important to take account of the correlations (between the parameter estimates), either in the analytical equations or in sampling from a multivariate distribution.

Ashley (1980), Lowe et al. (1982), Boyce (1999), Boyce and Bright (2003), Zhao and Kockelman (2002) and Beser Hugosson (2004, 2005) all study the problem of how a given transport model can not only produce a central estimate of traffic volume or revenues, but also uncertainty margins around these. The first five studies mentioned use Monte Carlo simulation for the inputs and for the parameter values instead of analytical methods. Zhao and Kockelman explicitly study the problem of propagation of errors: when a number of modules are used sequentially, errors can become bigger (reinforcing initial deviations) or smaller (equilibrium mechanisms). All these five studies use relatively simple aggregate transport models. In a paper on the Swedish national passenger transport model, Beser Hugosson (2004, 2005) uses the Bootstrap method on disaggregate mode-destination models, but leaves out input uncertainty, trip frequency models and congestion feedbacks. Analytical methods to calculate the uncertainty were not used.

Grue (1999) used random draws from multivariate normal distributions (with correlations) to derive output uncertainty due to model (parameter) uncertainty. The transport model was the disaggregate national transport model for Norway, with submodels for the possession of driving licences, car ownership, the number of trips (distinguishing long and short distances, including competition between modes). This application however did not include input uncertainty, destination choice models or congestion feedbacks.

How is uncertainty expressed?

Uncertainty in forecasted values can be expressed in many ways, but measures that were often used in the literature are:

- The variance of the forecast;
- Its standard deviation (square root of the variance);
- Its 95% confidence interval (-1.96 times the standard deviation to +1.96 times the standard deviation);
- Percentiles of its distribution, e.g. the lowest 1% or 5% for revenue or vehicle flow forecasts.

Order of magnitude of uncertainty

Many of the studies do not present quantitative outcomes and those that do are sometimes not comparable because they use different expressions. For link flows and total area-wide travel demand by mode studies find 95% confidence intervals of 5-14% of the mean (on both sides of the mean) for model uncertainty. This of course depends clearly on the sample size used in model estimation. Studies on input uncertainty or both model and input uncertainty obtain 95% confidence intervals for link flows between 18% and 33% of the mean. There are indications in the literature that input uncertainty is more important for uncertainty in traffic forecasts than model (parameter) uncertainty. For the value of time, different studies found 95% confidence intervals ranging from 6% to 48% of the mean value (depending especially on sample size).

Relation between the literature reviewed and the application presented in this paper

The literature either uses relatively simple aggregate transport models to generate output uncertainty, or (when it uses more complex models) only provides input uncertainty or model uncertainty. In our application to The Netherlands (see sections 3 and 4), we move beyond the literature reviewed, in that we provide output uncertainty in terms of numbers of tours, kilometres and vehicle flows for both model and input uncertainty, based on a relatively sophisticated disaggregate model system, including impacts through tour frequency, mode and destination choice, with and without congestion feedbacks. The lessons that we learnt from the literature on the choice of methodology are described in section 3.2.

3. METHOD FOR QUANTIFYING INPUT AND MODEL UNCERTAINTY IN THE DUTCH NATIONAL MODEL SYSTEM

3.1 A short introduction to the model

The remainder of this paper is about an application on quantifying uncertainty in traffic forecasts in The Netherlands. First, we shall briefly describe the transport model used, which is the Dutch national model system, LMS ('Landelijk Model Systeem'). The LMS was first developed in the 80's and has been used since for several policy documents on transport policy and for the evaluation of large transport projects (also see Gunn, 1999; Daly, 2000). It is a forecasting model for the medium to long term (the forecast year often being 20-30 years ahead), with a focus on passenger transport (freight traffic appears only in assignment of an exogenous OD truck matrix to the road network). It covers the whole of The Netherlands and some neighbouring areas, distinguishing more than 1,300 zones. The LMS consists of random utility submodels at the household or person level for:

- Licence holding, constrained to exogenous forecasts;
- Car ownership, constrained to exogenous forecasts;
- Tour frequency by travel purpose. A tour is defined as a round trip (e.g. home-work-shop-home). Here we distinguish eleven travel purposes. For each of these there is a model for the choice between zero tours[1] and one or more tours and a model for subsequent tours.
- Mode and destination choice: there are eight of these models, one for each of eight travel purposes. The modes distinguished are: car-driver, car passenger, train, bus/tram/metro, non-motorised.
- Departure time choice by travel purpose.

The model system is applied in a pivot-point fashion (Daly et al, 2005) whereby the demand models produce growth factors for the changes between the base year and forecasts year for each origin-destination relation by mode, purpose and time of day, and a given base matrix represents the traffic pattern in the base year. Then, the OD car driver demand matrices are assigned to the road network and after initial assignment there is a feedback to mode, destination and departure time choice (iterative application).

The models for which we study uncertainty in the LMS are the tour frequency models and the mode-destination choice models.

3.2 Conclusions on the choice of methodology from the review

For quantifying input uncertainty the most natural approach is Monte Carlo simulation. All the literature reviewed that dealt with input uncertainty used some variant of this method. We think it is important to include correlation between input variables (because some important ones are highly correlated, such as income and car ownership). Therefore, we decided to sample from a multivariate normal distribution for the input variables. How this distribution was derived is described below.

On the basis of the review of the literature we concluded that the preferred method for quantifying the model errors is either a combination of the Jack-knife/Bootstrap method to correct for specification error and Monte Carlo simulation for the uncertainty due to estimation, or the analytic method. The latter can only be used for relatively simple model specifications. We investigated whether analytic expressions could be used for the LMS tour frequency models and the mode-destination choice models. We derived the analytical expressions, but also found that very long run times would be required to evaluate these expressions. Therefore we decided to use Monte Carlo simulation for the model uncertainty as well.

3.3 Input uncertainty

First of all, a list of the most important autonomous variables influencing transport demand has been prepared (principally by going through the explanatory variables of the LMS tour frequency and mode-destination choice models, and the zonal targets in the population forecasting procedure QUAD[2]). This list does not include the policy variables that can also be found in these models (such as public transport fares, parking costs, the speeds of the transport modes), that can be influenced by users of the models (government at different levels, public transport operators). The sensitivity of the link flows to such variables is usually handled through policy sensitivity runs: changing one policy variable at a time, relative to a reference case, or by comparing the outcomes for different policy packages.

Licence holding is not included, as it is so high now (among the eligible age groups) that for the future only very limited variation is possible. The total amount of freight traffic, international traffic and the correction for changes in working hour practices were not varied in the simulations with LMS.

The list of the main autonomous forces for simulation of input uncertainty on transport demand (defined here as tour generation and mode-destination choice) is as follows:

- Household disposable income;
- Car ownership;
- Car cost per kilometre (only the fuel cost part, which is partly an autonomous and partly a policy variable, but not the toll and parking cost which are fully policy variables);
- Number of jobs (by sector), which serves as an attraction variable;
- Population by age group (or population and average age);

- Household size;
- Occupation (employed or unemployed by gender) and education (number of students per type of education);
- Part-time and full-time employment.

Time series data on these variables over a long period (1960-2000) was collected and analysed. For one variable (part-time working) no consistent data was available and in a few cases data was available only for 1970-2000. In the long run, periods of high growth and periods of low growth or even decline occur alternately (the business cycle). Therefore the income growth expectations for a 20-30 year horizon, which is likely to include a few of each, become smoothed. Other variables (number of jobs, number of cars, labour force) are also related to the business cycle. To express this phenomenon, we calculated 20-year moving averages (e.g. 1960-1979, 1961-1980, etc.). A time horizon of 20 years is not unusual for project evaluation. Often even longer periods (such as 30 years) are applied.

We used the standard deviations and correlations of the 20-year moving averages in the determination of the multivariate normal distribution from which the input values for LMS runs are to be drawn. The idea is that the amount of variation in the input variables over the next 20 years is determined from all 20-year moving averages over the past 40 years (an exception is car ownership for which we expect a trend towards saturation).

In order to determine the input variables for the simulation with the LMS, draws from a multivariate normal distribution were made. The Cholesky decomposition was used here as a method to generate a correlated multivariate normal distribution on the basis of uncorrelated univariate normal draws η (see for instance: Press et al., 1988). Multivariate random draws δ are then calculated using initial averages μ and the corresponding Choleski factor (matrix Λ). The Choleski factor expresses K correlated terms as arising from K independent components, with each component "loading" differently onto each term. For any pattern of covariance, there is some set of loadings from independent components that reproduces that covariance. Equation (1) shows the functional form for two variables, indexed 1 and 2.

$$\begin{aligned} \delta_1 &= \mu_1 + \Lambda_{11} \times \eta_1 \\ \delta_2 &= \mu_2 + \Lambda_{21} \times \eta_1 + \Lambda_{22} \times \eta_2 \end{aligned} \tag{1}$$

where

δ = the multivariate normal draw (vector)

μ = the initial average (vector)

Λ = the Choleski factor matrix

η = the random draw (vector) generated from a univariate normal distribution

The random number generator was replaced by Halton draws, which provides a better distribution (greater coverage, i.e. fewer empty spaces) over the ‘random’ space. Several Halton methods were tested initially. The shuffled Halton (see Hess, 2003) appeared to work best, and was selected.

The numbers drawn were then converted into LMS input variables and the models were run for these values. Every LMS run has a different set of values for model coefficients, explanatory variables and QUAD targets. All variables for which uncertainty is studied refer to national totals. In the LMS runs, the percentage changes in these national totals were applied to the zonal variables (e.g., all zones get richer by x %). This means that the relative distribution of these variables over the zones did not change.

Twenty draws were made for the input variables, which are used both for twenty LMS runs for the reference scenario and the twenty LMS runs with the new infrastructure project (see section 4), keeping the model coefficients constant. For the model coefficients (see section 3.4 below) again twenty draws were made, which were used in 40 runs (reference and project situation) as well, keeping the input variables constant. The first ten draws for input variables were further combined with the first ten draws for the model coefficients for reference and infrastructure scenario (twenty runs in total). This sums to a total of 100 LMS runs, 50 reference runs and 50 with the new infrastructure project.

3.4 Model uncertainty

In calculating the uncertainty around the link flows we focussed on the tour frequency models and the mode-destination choice models in the LMS. We did not include specification and estimation error in the licence holding and car ownership models (but treated the future year national car ownership total as one of the input variables to be varied, as described above). Similarly, we treated the parameters in the time of day choice models and the assignment (e.g. the speed-flow curves) as having been determined without error.

The general approach in this application therefore is that we study variations in the OD matrices (due to input variables and the tour frequency and mode-destination choice models) and assign these using the same assignment procedures, without introducing extra variation due to uncertainties (e.g. through Monte Carlo draws) in the departure and route choice functions. In this project we study the impact on the predicted flows for a set of selected links (three to four links in one direction). The assignment mechanism itself can change the amount of uncertainty (e.g. reduce it, as in Zhao and Kockelman, 2002), because larger transport demand leads to more congestion and this increases travel times, which reduces demand for specific routes, periods and modes, etc. This also implies that we had to run the full assignment procedure. The result of the first iteration were stored separately, to see the effect without the congestion feedback mechanism. Freight matrices for road transport were added to the OD matrices for cars that were varied in this project, but these freight matrices did not vary (fixed background vehicle loads).

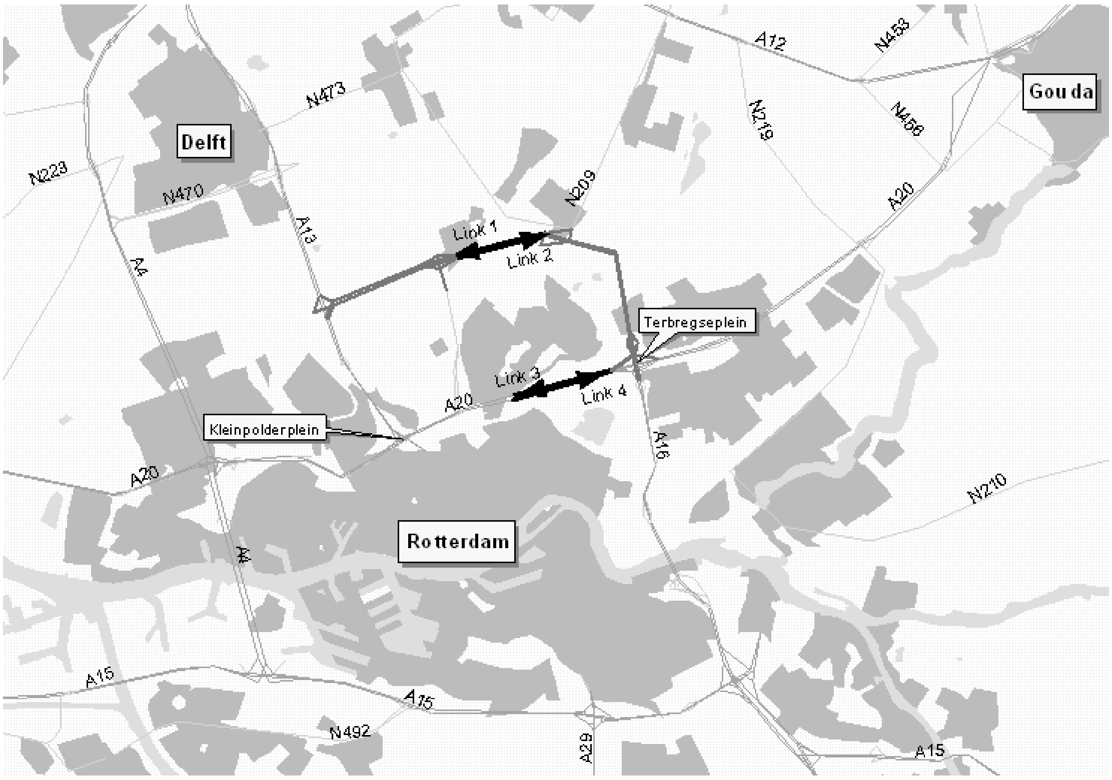
Using the Bootstrap method, we re-estimated all tour frequency models and the commuting mode-destination models. This slightly increased most of the standard deviations (decreased the t-ratios) of the tour frequency models, to reflect the additional uncertainty due to misspecification of the model (e.g. in the functional form, the independence and homoskedasticity assumptions on the error distribution, but also including misspecification due to omitted variables). The correlations between the parameter estimates were taken from the Bootstrap estimation as well. We found no systematic differences between the Bootstrap estimates of the commuting mode-destination model and the original estimates for this model (there were differences, usually small, in both directions). Therefore we used the standard deviations and correlations of the original estimation runs for the mode-destination models for all purposes other than commuting.

After this, the tool described in section 3.3 (Choleski decomposition) was used to generate twenty random draws from a multivariate normal distribution with this variance-covariance matrix. For the twenty-two tour frequency models (initial tour and multi-tour models) and the eight mode-destination models within the LMS the coefficients were changed by taking random draws from a multivariate normal distribution. For each of the twenty sets of coefficients, the models were run. Coefficients that have a fixed value (of one), such as structural coefficients for multinomial logit models and coefficients for the basic size (attraction) variables, have not been changed.

4. RESULTS FOR THE DUTCH NATIONAL MODEL SYSTEM

The project for which uncertainty in traffic forecasts is studied using the LMS is the extension of the A16 motorway in the Rotterdam area (see Figure 1). This concerns a major new road. At present, the A16 enters the Rotterdam area from the South (Breda, Dordrecht) and continues until the Terbregseplein, where it meets the A20. The extension would continue north of the A20, and after a few kilometres it would go west until it meets the A13. This new road has three new links in the LMS, with several access links. It would form an alternative for several existing routes, but especially for the A20 between the Kleinpolderplein and the Terbregseplein.

Figure 1. The road network in the Rotterdam area in 2020 (with thick lines indicating the new road project, and arrows the links selected for analysis).



The uncertainty of the LMS forecasts was studied both at the national level and at link level:

- National level: number of tours and passenger kilometres, by mode and purpose, irrespective of whether this takes place on the network or not. This is output from the mode-destination component of the LMS.
- Link level: traffic flow (in passenger car equivalent units), travel times (hours) and vehicle hours lost on a number of selected links: links on the new road (links 1 and 2 in Figure 1), some directly affected links (links 3 and 4) and a sort of 'control' link.

In total, 100 LMS runs were carried out, as indicated in the previous section:

- 50 runs for the reference 2020;
- 50 runs for the reference 2020 with the extended A16 project.

Each set of 50 runs consists of

1. 20 runs for variation in the model input variables;
2. 20 runs for variation in the model coefficients;
3. 10 runs with variation in both model input variables and coefficients.

In Table 2 are results for the Reference 2020 simulation with the LMS tour frequency and mode destination models, at the national level by mode for the sum of all purposes.

Table 2. Effect of input uncertainty and model uncertainty on the predicted number of tours for the reference situation.

	Mean (millions of tours per day)	Standard deviation (as % of mean) due to input uncertainty keeping model coefficients constant	Standard deviation (as % of mean) due to model uncertainty keeping model inputs constant	Standard deviation (as % of mean) due to model and input uncertainty
Car driver	11.7	11.7%	0.8%	12.1%
Car passenger	3.9	6.3%	0.9%	6.0%
Train	0.7	15.3%	2.4%	16.2%
Bus/tram/metro	0.7	12.2%	1.4%	12.1%
Slow modes	15.4	4.1%	0.5%	4.3%
Total	32.4	1.8%	0.6%	1.9%

In assessing the input error margins, one should keep in mind the amount of variation in the input variables that was introduced in the LMS runs. The average income increase of 65% between 1995 and the reference 2020 for instance was varied from a 13% increase to a 110% increase.

The standard deviations in the numbers of tours that are due to input variation at this level are between 4% and 16% (by mode). For the total number of tours over all modes, the standard deviation is even below 2%. The total across all modes does not include the distribution over modes from the mode-destination models, and therefore it can be predicted more precisely.

The standard deviations that result from model uncertainty are clearly smaller than for input uncertainty. This happens for all of the modes and for the total over modes. For instance for car drivers this standard deviation is only 0.8% of the mean for tours, and for all modes together it is 0.6%. Uncertainty in the input variables such as income and car ownership clearly dominates the uncertainty that is due to the uncertainty in the model coefficients. The amount of model uncertainty we find here for the LMS is clearly smaller than the 4%-7% (standard error as % of the mean) that Beser Hugosson (2004, 2005) found in Sweden for total demand by mode, only using mode-destination choice (no impact on tour frequencies, no congestion feedback). We think that the main reason for the relatively low model errors is the large sample that was used in estimation of the LMS (the national travel survey used contains trip diary information for more than 160,000 persons). Grue (1999) also found for the Norwegian national model that the confidence intervals for model (parameter) uncertainty were ‘rather small’. This conclusion however was based on an investigation of the impacts of variation in a set of coefficients (either income or transport time and costs) at a time, not simultaneous variation of all important coefficients as in our application.

As can be seen in Table 2, train and bus/tram/metro use are more uncertain than the use of car and the non-motorised modes. The model uncertainty is higher, probably because there are fewer observations in the estimation data for these modes and the relevant coefficients are therefore less well estimated. Also the input uncertainty is higher for public transport: the use of public transport is more sensitive to how exogenous variables, such as income and the composition of the population, will develop than the use of other modes.

For car driver kilometres (see Table 3), the standard deviation due to input uncertainty is 8.3% of the mean. The 95% confidence interval from the Normal is from 287 million to 399 million car kilometres. For car drivers, the relative input uncertainty for passenger kilometres is smaller than for tours. For car passenger and the slow modes however, the reverse in true.

Table 3. Effect of input uncertainty and model uncertainty on the predicted number of passenger kilometres for the reference situation.

	Mean (millions of kilometres per day)	Standard deviation (as % of mean) due to input uncertainty keeping model coefficients constant	Standard deviation (as % of mean) due to model uncertainty keeping model inputs constant	Standard deviation (as % of mean) due to model and input uncertainty
Car driver	343	8.3%	0.7%	8.3%
Car passenger	107	10.2%	3.9%	11.0%
Train	71	14.4%	2.5%	15.2%
Bus/tram/metro	21	10.4%	2.1%	10.0%
Slow modes	80	4.7%	0.5%	5.0%
Total	622	4.4%	0.9%	4.5%

It is an interesting outcome that the errors in the kilometres are of the same order of magnitude as the errors in the numbers of tours, while for policy simulations that change travel times and costs, the kilometres are usually more volatile than the tours (e.g. greater time and cost elasticities for kilometres). Since this happens for all modes, the explanation cannot (only) be the effect of congestion (that would dampen the kilometrage shifts). We conclude that runs that change time and cost affect destination choice more than mode choice, and tour frequencies not at all[3]. The changes in the input variables performed here (of which income and car ownership are the most important) affect tour frequency and mode choice more than destination choice, and therefore lead to broadly similar effects in terms of tours and kilometres.

In most cases the standard deviations for input and model uncertainty in tours or passenger kilometres are slightly higher than those for input uncertainty alone, but in some cases the standard deviations for both sources of uncertainty are the same or just below those for input uncertainty. This is probably an artefact of having performed only a limited number of LMS runs for the combination of the two sources of uncertainty. But, as for tours, the input uncertainty in the number of passenger kilometres is substantially greater than the model uncertainty.

For the situation with the project, the variation in tours and kilometres is of the same order of magnitude as for the reference situation. Again, the uncertainty due to input variation dominates the output variation. The relative uncertainty around the difference in total car tours or kilometres (with and without the road project) from the tour frequency and mode-destination models is much larger than the above-mentioned results, but this concerns very small amounts of traffic (at the national scale).

We also had a look at the simulation results at the national level for the reference 2020 from the tour frequency and mode-destination models, without congestion feedback (congestion travel times are led back to the travel demand choice models), to see whether the congestion feedback leads to a damping of the variation on tours and kilometres or to a propagation of errors. We concluded that the uncertainty in the number of tours is the same with and without congestion feedback, and that with congestion feedback the variation in kilometres is slightly smaller.

In Table 4 are the key outcomes for the vehicle flow in passenger car equivalent units at selected links (in the reference we only have three links, in the situation with the project we have five, but in Table 4 for the project situation we only present results for the new link in both directions). This variable refers to a full 24-hours day, not just to the peak hours.

Table 4. Effect of input uncertainty and model uncertainty on the predicted vehicle flows.

Selected link (between brackets: link number in Figure 1)	Mean (number of vehicles per day*1,000)	Standard deviation (as % of mean) due to input uncertainty keeping model coefficients constant	Standard deviation (as % of mean) due to model uncertainty keeping model inputs constant	Standard deviation (as % of mean) due to model and input uncertainty
Reference situation:				
A20 Rotterdam-Gouda (link 4)	83	4.1	0.3	4.3
A20 Gouda-Rotterdam (link 3)	87	4.6	0.6	4.7
A2 Amsterdam-Utrecht	115	8.3	1.3	8.3
Project situation:				
New link (A16) Rotterdam-Delft (link 1)	38	11.9	1.2	12.3
New link (A16) Delft-Rotterdam (link 2)	30	14.8	6.4	15.7

The standard deviations of the link flows in the reference situation are between 4% and 9% for input uncertainty, and around 1% for model uncertainty. Beser Hugosson (2004, 2005) found 4%-6% for the impact of model error on link flows in Sweden. For the number of hours travelled, the standard deviations on the A20 and A2 are between 4 and 13% for input variation and 1-3% for model uncertainty. Again input uncertainty clearly dominates model uncertainty. The number of hours lost due to congestion, labelled ‘Q-hours’, can have a much larger uncertainty, especially the relative uncertainty can be high when the absolute numbers of Q-hours are low.

The standard deviations for the differences between the situation with and without the road project in link flows for links competing with the new road are 8-12% for input uncertainty, 5-6% for model uncertainty and 7-13% for combined uncertainty. So for differences between the situation with and without the project, the model errors are relatively more important than for the absolute traffic forecasts, where the input errors dominate the picture. Again the Q-hour differences are very uncertain, and the difference in hours travelled are in between. With regards to the evaluation of the project (the A16 extension in this example): the flow on this new link is predicted with a substantial level of uncertainty: the link flows can be up to 30% higher or lower than in the most likely case (also see Table 4, bottom two rows). This means that for cost-benefit analysis of the project, relatively large variations in the benefits need to be evaluated to account for uncertainty in the inputs and models.

We also carried out another case study on quantifying uncertainty for a regional model. Here we used the NRM (New Regional Model, a model derived from and similar in general structure and coefficients to the LMS) Noord-Brabant (with permission from the Regional Directorate, who own this model system, and kindly co-operated in providing the appropriate input files). As for the LMS application, we selected a road project, in this case the Eindhoven eastern ring road (‘Oostelijke Randweg’), that would complete the beltway around the city of Eindhoven. The outcomes in terms of relative uncertainty are generally speaking quite similar to those of the application of the LMS at the national level.

5. CONCLUSIONS AND RECOMMENDATIONS

In this paper we presented a review of the literature on uncertainty in traffic forecasts, and a new application in The Netherlands. The key outcomes are summarised below.

Review of the literature

We found that the literature on quantifying uncertainty in traffic forecasts is fairly limited. We distinguished between input uncertainty (e.g. on the future incomes and car ownership levels) and model uncertainty (including specification error and error due to using parameter estimates instead of the true values).

For quantifying the amount of input uncertainty all contributions that we found in the literature use some form of repeated model simulation (sensitivity testing). Usually statistical distributions are postulated for the input variables and then random draws are made from these distributions. This generates input values that are used subsequently in model runs. The uncertainty is calculated from the variance over all the runs for the different input values.

Most studies apply univariate distributions for the input variables (ignoring correlation between inputs).

Several methods have been found in the literature for quantifying model uncertainty in transport forecasts. A few studies used analytic expressions for the variance of the endogenous variable that results from using parameter estimates for the influence of the exogenous variables. For complicated models, these expressions become very cumbersome. The Jack-knife and Bootstrap method can be used to obtain proper t-ratios or standard errors for the model coefficients in situations with specification error (such as repeated measurements in panel and SP data). These more correct standard errors of the parameters can be used either in the analytic calculation of the standard error of the model outcomes or as information on the statistical distributions from which values can be drawn for model simulation runs. Again, it is important to take account of the correlations (between the parameter estimates).

Development of a method for LMS

In our analysis of uncertainty in traffic forecasts from the Dutch national model system LMS, we used existing time series as the key source of information on means, standard deviations and correlations of input variables, and applied these to get multivariate distributions for the model input variables, to account for correlation between the input variables.

Analytic methods to quantify the model uncertainty were considered and the analytic expressions were worked out, but the evaluation of these expressions would take too much computer time. For quantifying the model errors we used the Bootstrap method to correct for specification error and Monte Carlo simulation for the uncertainty due to estimation, for the tour frequency and mode-destination choice models in the LMS.

Outcomes for LMS

Both the input variables and the model coefficients of the LMS tour frequency and mode-destination models were varied. Half of the model runs were for the reference situation 2020, the other half for the situation with a specific road project (extension of the A16 near Rotterdam).

We found substantial, but not very large, uncertainty margins for the total number of tours and kilometres (by mode) in the study area of the LMS and for the vehicle flows on selected links. The uncertainty margins for differences between a project and a reference situation are not much larger, unless these differences are of a small magnitude. In many cases, there is greater variation in the number of hours lost due to congestion than in hours travelled. The contribution of input uncertainty (e.g. in future incomes or car ownership levels) to these errors is generally much larger than that of model uncertainty (e.g. coefficients estimated with some error margin).

These outcomes for uncertainty in traffic forecasts include variation in most of the input variables for the LMS travel frequency and mode-destination choice models, as well as the error in these models. Sources of variation that were not included are:

- Uncertainty in the base matrices, which are combined with model outcomes for a base year and a future year to obtain forecasts for the future year[4].
- Errors in the licence holding and car ownership models (note however that errors in the total number of cars were included in the input variation).
- Errors in the assignment and time-of-day procedures. These models were used in the LMS runs carried out (for different demand forecasts from the tour frequency and mode-destination models) but without varying their parameters.
- Uncertainty due to a different distribution over zones. In our simulations we applied the same proportional change for some variable in each zone.
- Uncertainty about the distribution of workers between part-time and full-time workers.
- Because in our method for quantifying uncertainty we relied on the long-run equilibrium models LMS, we were not able to present the time path of the uncertainty estimates, but only final 2020 outcomes. Nevertheless, especially for PPP projects, the returns in the first years and the uncertainty attached to these are often very important. This would require dynamic models.

The distribution over zones can to some degree be incorporated in scenario studies, where different zonal distributions can be postulated. Scenario studies however do not include probabilities for the variables and future states that they describe and can therefore not be used to calculate uncertainty margins. Our study overlaps to some degree with a scenario approach in that both methods try to include correlations between attributes that characterise the future state. We went beyond scenarios by using a specific probabilistic approach so that we could produce quantitative uncertainty estimates. On the other hand a scenario approach could complement the approach used here, because it offers a way to include varying assumptions on the zonal distribution (e.g. of incomes). Conversely, the probabilistic simulation approach using information from past time series on input variables (including correlations) could also be used in the generation of scenarios, by selecting a limited number of settings for the input variables from the simulations (e.g. one intermediate, one where factors influencing demand for travel take on low values and one where the factors take high values).

The method for quantifying uncertainty that was developed in this paper can be used in the assessment of proposed transport projects where the LMS is used to provide the traffic demand changes. But since the method is very computer-intensive (requiring 100 model runs; a smaller number of runs would not be acceptable), this will only be feasible for the evaluation of major transport projects. For other projects, the quantitative outcomes for the applications presented in this paper can provide guidance.

Notes

1. For non-home-based purposes, trip frequencies are modelled rather than tour frequencies; mode and destination choice are also modelled at trip level for these purposes.
2. There is a computer routine QUAD (from 'quadratic') within the LMS that produces the joint distribution of socio-economic attributes of the households, given the total population and the marginal distributions for these attributes, both from external sources. This routine is based on quadratic optimisation, following Daly (1998).
3. The fact that changes in time and cost do not affect tour frequencies is simply due to model specification: the tour frequency models do not include an 'accessibility' effect.
4. The LMS base matrices were estimated on multiple data sources using formal maximum likelihood methods. This means that standard deviations for matrix uncertainty should be available and could be used in simulation methods to include uncertainty from the base matrices.

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